Minimal generating set and structure of wreath product of cyclic groups, comutator of wreath product and the fundamental group of orbit Morse function $\pi_1 O(f)$

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Let i_j be the orders of C_{i_j}. In this work the previous result of the author [1] is strengthen also there is considered new class of *wreathcyclic* groups $\Im$ (let $G \in \Im$) which constructed by formula:

$$G = (\underset{j_0=0}{\overset{n_0}{\wr}} C_{k_{j_0}}) \times (\underset{j_1=0}{\overset{n_1}{\wr}} C_{k_{j_1}}) \times \ldots \times (\underset{j_l=0}{\overset{n_l}{\wr}} C_{k_{j_l}}), 1 \leq k_{j_i} < \infty, n_i < \infty.$$

Theorem 1. *If orders of cyclic groups $\mathbb{C}_{n_i}$, $\mathbb{C}_{n_j}$ is mutually coprime $i \neq j$ then the group $G = C_{i_1} \wr C_{i_2} \wr \ \ldots \ \wr C_{i_m}$ admits two generators β_0, β_1.*

The subtree of X^* induced by the set of vertices $\cup_{i=0}^{k} X^i$ is denoted by $X^{[k]}$.

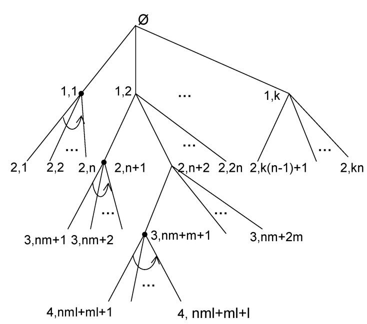

Figure 1.1. Directed automorhism

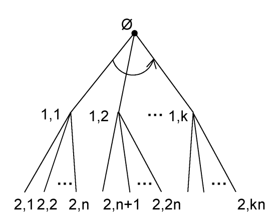

Figure 1.2. Rooted automorhism

We construct the generators of $\underset{j=0}{\overset{n}{\wr}} C_{i_j}$ as a rooted automorphism β_0 in Figure 2 and a directed β_1 along a path l in Figure 1.1 on a rooted labeled truncated tree $X^{[k]}$.

Let $l = x_1 x_2 x_3 ... x_k$ be an finite ray in $X^{[k]}$.

Definition 2. We say that the automorphism g of $\mathbb{X}$ is directed along l and we call l the spine of g if all vertex permutations along the ray l and all vertex permutations corresponding to vertices whose distance to the ray l is at least 2 are trivial (Figure 1).

Definition 3. An automorphism of X is rooted if all of its vertex permutations that correspond to non-empty words are trivial.

Corollary 4. *A center of the group* $\mathbb{Z}\ltimes_{\phi}(\mathbb{Z})^{n} \simeq (\mathbb{Z}, X) \wr \mathbb{Z}$ *consists of normal closure of diagonal of* Z^n*, trivial an element, and kernel of action by conjugation that is* $n\mathrm{Z}$*. Other words*

$$Z(H) = \langle 1;\ \underbrace{h, h, ..., h}_{n}),\ e,\ (n\mathbb{Z}, X) \wr \mathbb{E}\rangle \simeq n\mathbb{Z} \times \mathbb{Z},$$

where $h,\ g \in \mathrm{Z}$, $Z(H) \simeq n\mathbb{Z} \times \mathbb{Z}$.

Corollary 5. *A center of a group of form* $\mathbb{Z}\ltimes_{\phi}(\mathcal{B})^{n} \simeq (\mathbb{Z}, X) \wr \mathcal{B}$ *generates by normal closure of: diagonal of* $\mathcal{B}^n$*, trivial an element, and* $n\mathrm{Z} \underset{X}{\wr} \mathcal{E}$.

In our case the Morse function [2] f on M that has the following properties:

1. f is constant on the bound M,
2. it has 2 points of maximum at a saddle point,
3. at these 2 points of maximum, the values of the function are equal; in every critical point of f the germ of f is C^{∞} equivalent to some homogeneous polynomial of 2 real variables without multiple factors.

Consider a group H of automorphisms of M which are induced by the action of diffeomorphisms h of a group $D(M)$ such that preserving the Mebius function f, that is, such h are from the stabilizer $S\,(f) \triangleleft D(M)$. Generators of their stabilizers by right action by diffeomorphisms $\pi_0 S(f|_{X_i}, \partial X_i)$ are τ_i.

The first generator ρ of cyclic group Z realizes shift of Mebius band and second τ realize rotation of domains X_i of simple connectedness on Mebius band when passing through the twisting point of Mebius band (M).

Proposition 6. *The group* $H \simeq \mathbb{Z}\ltimes(\mathbb{Z})^{n} = \langle \rho,\ \tau\rangle$ *with defined above homomorphism in* $AutZ^n$ *has two generators and non trivial relations*

$$\rho^{n}\tau\rho^{-n} = \tau^{-1},\ \rho^{i}\tau\rho^{-i}\rho^{j}\tau\rho^{-j} = \rho^{j}\tau\rho^{-j}\rho^{i}\tau\rho^{-i},\ 0 < i, j < n.$$

Also this group admits another presentation in generators and relations

$$\left\langle \rho, \tau_1, ..., \tau_n \,\middle|\, \rho\tau_{i(\bmod n)}\rho^{-1} = \tau_{i+1(\bmod n)}\,,\ \ \tau_i\tau_j = \tau_j\tau_i,\ i,\ j \leq n \right\rangle. \tag{1}$$

Proposition 7. *The commutator of Sylow 2-subgroup* $(Syl_2 A_{2^k})'$ *has order* 2^{2^k-k-2}.

Proposition 8. *The second commutator of Sylow 2-subgroup* $(Syl_2 A_{2^k})$ *has the order* 2^{2^k-3k+1}.

Corollary 9. *The Frattini factor of* $(Syl_2 A_{2^k})'$ *is isomorphic to elementary abelian subgroup* $(C_2)^{2k-3}$*. Any minimal generator set of* $(Syl_2 A_{2^k})'$*. has* $2k-3$ *generators.*

Example 10. The minimal generating set of $Syl_2'(A_8)$ consists of 3 generators: $(1,3)(2,4)(5,7)(6,8)$, $(1,2)(3,4), (1,3)(2,4)(5,8)(6,7)$. The commutator $Syl_2'(A_8) \simeq C_2^3$ that is an elementary abelian 2-group of order 8. Minimal generating set of $Syl_2'(A_{16})$ consist of 5 (that is $2 \cdot 4 - 3$) generators:
$(1,4,2,3)(5,6)(9,12)(10,11), (1,4)(2,3)(5,8)(6,7), (1,2)(5,6)$,
$(1,7,3,5)(2,8,4,6)(9,14,12,16)(10,13,11,15), (1,7)(2,8)(3,6)(4,5)(9,16,10,15) \times$
$\times\ (11,14,12,13)$.

References

[1] *R. V. Skuratovskii,* Minimal generating sets for wreath products of cyclic groups, groups of automorphisms of Ribe graph and fundumental groups of some Morse functions orbits. (in Russian) Summer school *Algebra, Topology and Analys*, P. 121–123, 2016.

[2] *S. I. Maksymenko,* Deformations of functions on surfaces by isotopic to the identity diffeomorphisms. 2013, arXiv:1311.3347.